An Improved Adaptive Kalman Filter for In-motion Initial Alignment of GPS-Aided SINS

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Abstract. In GPS-aided strap-down inertial navigation system, in-motion initial alignment is crucial and can be solved with a closed-loop scheme based on state estimation. With this method, the noise covariance matrices need to be estimated, which, however, can be inaccurate in practice. In this paper, a novel adaptive Kalman filter is proposed to address the above problem. The state and measurement noise covariance matrices are jointly estimated based on a variational Bayesian approach, in which the prior and posterior probability density functions of the state noise covariance matrix and one-step prediction error covariance matrix are assumed to have the same form. Simulation results demonstrate that the proposed algorithm can improve the initial alignment accuracy of the in-motion initial alignment based on a closed-loop scheme as compared with an existing baseline adaptive Kalman filter.

Keywords: Initial alignment, adaptive Kalman filter, strap-down inertial navigation system, closed-loop, variational Bayesian

1 Introduction

Strap-down inertial navigation system (SINS) has been used in many applications because of its excellent properties, such as independence and sigma-completeness [1]. To guarantee the navigation accuracy of SINS, the initial alignment is necessary before navigation process starts [2]. Many methods have been proposed for initial alignment, which, however, can only be used on the static base or swaying base [3]. With the expansion of navigation applications, these methods are not suitable for in-motion initial alignment.

To address this problem, a series of optimization-based alignment (OBA) methods has been proposed by Wu et al [4]–[5], in which the idea of attitude determination method was firstly used to solve the in-motion initial alignment problem. However, the OBA method suffers from two problems: one is that the sensor bias cannot be estimated; and the other is that the error accumulations are introduced from sensor errors by the integral calculation in the calculations of observation vectors [6]. To solve these problems, a closed-loop attitude estimate-based initial alignment method (CLAEBIA) which can estimate and compensate the gyroscope drift in real time to improve the precision of the state-space model and accelerate the filtering convergence speed has been proposed by Huang et al [7]. As an extension, Huang et al. have proposed an improved closed-loop attitude estimate-based initial alignment (ICLAEIA) method to take more comprehensive error factors into consideration [8]–[9]. The ICLAEIA method builds a linear state-space model by considering the gyroscope drift error, accelerometer bias error and GPS lever arm error as the state variables. Therefore, the ICLAEIA method improves the speed and accuracy of initial alignment by compensating these error factors. However, for these CLAEBIA methods, the measurement noise covariance matrix (MNCM) cannot be determined due to the use of multiple sensor outputs and previous estimated parameters during the construction of the measurement equation. Meanwhile, the state noise covariance matrix (SNCM) is related to the coupling term of gyroscope random noise and unknown state variable. Therefore, the SNCM cannot be determinated. The accuracy of the traditional Kalman filter algorithm will decrease significantly and even lead to the divergence of filtering results when the inaccurate SNCM and MNCM are used.

Up to now, the mainstream method for solving the estimation problem with inaccurate noise covariance matrices is the use of adaptive Kalman filters (AKFs). Husa et al. proposed a Sage-Husa AKF (SHAKF) [10], which is a recursive method to estimate the mean and covariance of state and measurement noises by using innovation and the weighted average method with exponentially fading memory. However, the SHAKF cannot guarantee convergence to the correct noise mean and covariance matrix. Mohamed et al. have proposed an innovation-based AKF (IAKF), which is based on a maximum likelihood criterion [11]. The maximum likelihood estimate of the innovation covariance matrix is calculated by constructing the measurement likelihood function in a sliding window to estimate the unknown noise parameters. The IAKF is not suitable for rapidly varying noise and requires a large data window to estimate noise mean values and covariance matrices. Sarkka et al. proposed an AKF based on the variational Bayesian approach (VBAKF-R) to estimate an inaccurate and slowly varying MNCM [12]. The VBAKF-R is sensitive to the influence of the inaccurate SNCM, which may lead to the decrease in filtering accuracy. Huang et al. proposed a novel AKF based on the variational Bayesian approach (VBAKF-PR) [13]. Accroding to the VBAKF-PR, the inaccurate SNCM only affects the inaccurate one-step predicted error covariance matrix (PECM). The inaccurate one-step PECM and MNCM are estimated jointly based on variational Bayesian approach. However, the VBAKF-PR method is sensitive to the selections of the nominal SNCM, tuning parameter, forgetting factor and nominal initial estimation error covariance matrix.

In this paper, a novel AKF based on the variational Bayesian approach (VBAKF-PRQ) is proposed to handle the problem mentioned above. As a heuristic idea, the SNCM is updated indirectly by using the estimated one-step PECM according to the time-update of the one-step PECM in the Kalman filter, although the SNCM cannot be updated directly under the Bayesian estimation framework. Firstly, the prior and posterior probability density function (PDF) of the SNCM are modeled as an inverse Wishart (IW) PDF with the same function form as the prior and posterior PDF of the one-step PECM. Then, the state vector, one-step PECM and MECM are jointly estimated by using the variational Bayesian approach. Finally, the posterior PDF of the SNCM is updated indirectly by using the estimated one-step PECM. Simulation results confirm that the proposed VBAKF-PRQ has advantages as compared with the existing VBAKF-PR method in terms of initial alignment accuracy.

The remainder of this paper is divided into four parts. In Section 2, the ICLAEIA method is analyzed, and the shortcoming of the VBAKF-PR algorithm is briefly explained. In Section 3, the main idea of the VBAKF-PRQ algorithm is derived. In Section 4, the method is verified by simulations. In Section 5, conclusions and future work are drawn.

2 Problem Statement

In this section, a brief overview of the existing ICLAEIA method is given, and the limitations of this method are analyzed. Furthermore, the shortcoming of the existing VBAKF-PR algorithm is briefly discussed. The existing ICLAEIA method is mainly divided into two steps in each iteration [8]. The first step is to calculate the attitude matrix at the initial moment of initial alignment by using the attitude determination approach. The second step is to update the linear state-space model and estimate the state vector. Then the estimated state vector is used to compensate the sensor error and the calculation error. In this paper, we mainly focus on the state estimation problem of the linear state-space model under the unknown SNCM and MNCM. Therefore, it is worth briefly reviewing the linear state-space model of the ICLAEIA method.

Firstly, the state vector is defined as $\mathbf{X} = \left[\varphi^{\mathrm{T}} \;\; (\delta\varepsilon^b)^{\mathrm{T}} \;\; (\delta\nabla^b)^{\mathrm{T}} \;\; (\delta\mathbf{l}^b)^{\mathrm{T}}\right]$, where φ, $\delta\varepsilon^b$, $\delta\nabla^b$ and $\delta\mathbf{l}^b$ denote respectively the misalignment angle of the attitude matrix $\mathbf{C}_{b(t)}^{b(0)}$, the gyroscope drift error, accelerometer bias error and lever arm error.

The differential equation of the misalignment angle is written as [8]

$$\dot{\varphi} = -\bar{\omega}_{ib}^{b} \times \varphi + \delta\varepsilon^b + \eta_\varphi \tag{1}$$

where $\bar{\omega}_{ib}^{b}$ is the gyroscope output value with the closed-loop compensation, η_φ is the noise of misalignment angle which is related to the gyroscope random noise, and the gyroscope drift error $\delta\varepsilon^b$ is established as a random walk process. The differential equation of $\delta\varepsilon^b$ can be obtain as

$$\delta\dot{\varepsilon}^b = \eta_g \tag{2}$$

where η_g is Gaussian white noise with zero mean. Moreover, $\delta\nabla^b$ and $\delta\mathbf{l}^b$ are both random constants.

According to (1)–(2), the state equation of the ICLAEIA method is as follows

$$\dot{\mathbf{X}} = \mathbf{\Phi}\boldsymbol{X} + \mathbf{w} \tag{3}$$

where the state noise $\mathbf{w} = \left[(\eta_\varphi)^{\mathrm{T}} \;\; (\eta_g)^{\mathrm{T}} \;\; \mathbf{0}_{1\times3} \;\; \mathbf{0}_{1\times3}\right]^{\mathrm{T}}$, and the continuous-time state one-step transition matrix $\mathbf{\Phi}$ is given by [8]

$$\mathbf{\Phi} = \begin{bmatrix} \left(-\bar{\omega}_{ib}^{b}\times\right) & \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{bmatrix} \tag{4}$$

where $(\vartheta\times)$ denotes the skew symmetric matrix of the vector ϑ. The continuous-time equation of the state vector needs to be discretized for numerical calculations, and the discrete-time state equation is given by

$$\mathbf{X}_k = \mathbf{F}_k\mathbf{X}_{k-1} + \mathbf{w}_{k-1} \tag{5}$$

where discrete-time state transition matrix $\mathbf{F}_k$ is

$$\mathbf{F}_k = \mathbf{I}_{12} + \mathbf{\Phi}T_s \tag{6}$$

The measurement equation of the ICLAEIA method is as follows [8]

$$\mathbf{Z}_k = \mathbf{H}_k\mathbf{X}_k + \nu_k \tag{7}$$

where the measurement vector $\mathbf{Z}_k$ is formulated as

$$\mathbf{Z}_k = \beta_v(t) - \mathbf{C}_b^n(0)\mathbf{C}_{\hat{b}(t_k)}^{b(0)}\tilde{\alpha}_{\mathrm{V}1}(t) - \tilde{\alpha}_{\mathrm{V}2}(t)\hat{\nabla}_{k-1|k-1}^{b} - \tilde{\alpha}_{\mathrm{V}3}(t)\hat{\mathbf{l}}_{k-1|k-1}^{b} \tag{8}$$

and t denote the current time. $\hat{\nabla}^b_{k-1|k-1}$ and $\hat{\mathbf{l}}^b_{k-1|k-1}$ represent the compensated accelerometer bias and compensated lever arm, respectively. The calculations of the observation vectors $\beta_v(t)$, $\tilde{\alpha}_{V1}(t)$, $\tilde{\alpha}_{V2}(t)$ and $\tilde{\alpha}_{V3}(t)$ are given by

$$\begin{aligned}\beta_v(t) = &\mathbf{C}^{n(0)}_{n(t)}\mathbf{V}^n_{gps}(t) - \mathbf{C}^{n(0)}_{n(t_m)}\mathbf{V}^n_{gps}(t_m) \\ &+ \sum_{k=1}^{M-1} \mathbf{C}^{n(0)}_{n(t_k)} \left[T_s\mathbf{I_3} + \tfrac{1}{2}T_s^2\omega^n_{in}(t_{k+1})\times\right]\left[\omega^n_{ie}(t_{k+1}) \times \mathbf{V}^n_{gps}(t_{k+1}) - \mathbf{g}^n(t_{k+1})\right]\end{aligned} \tag{9}$$

$$\tilde{\alpha}_{V1}(t) = (\mathbf{C}^{b(0)}_{\hat{b}(t_k)})^T \sum_{k=1}^{M-1} \mathbf{C}^{b(0)}_{\hat{b}(t_k)} \left[T_s\mathbf{I_3} + \frac{1}{2}T_s^2\bar{\omega}^b_{ib}(t_{k+1})\times\right] \tilde{\mathbf{f}}^b(t_{k+1}) \tag{10}$$

$$\tilde{\alpha}_{V2}(t) = -\mathbf{C}^n_b(0) \sum_{k=1}^{M-1} \mathbf{C}^{b(0)}_{\hat{b}(t_k)} \left[T_s\mathbf{I_3} + \frac{1}{2}T_s^2\bar{\omega}^b_{ib}(t_{k+1})\times\right] \tag{11}$$

$$\tilde{\alpha}_{V3}(t) = \mathbf{C}^n_b(0)\left(\mathbf{C}^{b(0)}_{\hat{b}(t)}\left[\bar{\omega}^b_{ib}(t)\times\right] - \mathbf{C}^{b(0)}_{\hat{b}(t_m)}\left[\bar{\omega}^b_{ib}(t_m)\times\right]\right) \tag{12}$$

where $t - t_m = MT_s$ with M and T_s denoting the sliding-window-size and the sampling time of IMU. $\tilde{\mathbf{f}}^b(t_{k+1})$ denotes the measured specific forces. The measurement matrix $\mathbf{H}_k$ is given by [8]

$$\mathbf{H}_k = \left[\mathbf{C}^n_b(0)\mathbf{C}^{b(0)}_{\hat{b}(t_k)}\tilde{\alpha}_{V1}(t) \;\; \mathbf{0}_3 \;\; \tilde{\alpha}_{V2}(t) \;\; \tilde{\alpha}_{V3}(t)\right] \tag{13}$$

It can be found from (1) that the state noise of the misalignment angle is mainly related to the coupling term of gyroscope random noise and unknown state vector, which leads to unknown SNCM. Meanwhile, it can be found from (8)–(12) that the MNCM cannot be determined due to the use of multiple sensor outputs and the previous estimated parameters during the construction of the measurement equation. Under the use of inaccurate SNCM and MNCM, the estimation performance of the traditional Kalman filter algorithm will degrade significantly, which further leads to the decrease of initial alignment accuracy.

As an advanced AKF algorithm, VBAKF-PR has been used in state estimation problem for a linear state-space model with inaccurate SNCM and MNCM. In the VBAKF-PR algorithm, the state vector, the one-step PECM and the MNCM are jointly estimated base on the variational Bayesian approach. However, the estimation accuracy of the one-step PECM is largely dependent on the selection accuracy of the nominal initial estimation error covariance matrix and nominal SNCM. The estimation precision of the one-step PECM will decrease when the inaccurate nominal initial estimation error covariance matrix and the inaccurate nominal SNCM are used. As a result, the estimation performance of the VBAKF-PR also degrades substantially.

3 Proposed VBAKF-PRQ

To solve the problem mentioned above, a novel VBAKF algorithm is proposed and applied in the in-motion initial alignment, in which the state vector, SNCM and MNCM are jointly estimated. For the novel VBAKF-PRQ algorithm, the estimation process of the state vector and MNCM is the same as that of the VBAKF-PR algorithm, and both are estimated based on the VB approach [13]. As a heuristic idea, the SNCM is assumed to have the same prior and posterior PDFs as the one-step PECM. Then, the SNCM can be updated indirectly by using the estimated one-step PECM according to the time-update of the one-step PECM in the proposed VBAKF-PRQ algorithm, since the SNCM cannot be updated directly under the Bayesian estimation framework. Firstly, according to time-update of the Kalman filter, the one-step PECM is formulated as

$$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^T + \mathbf{Q}_{k-1} \tag{14}$$

According to (14), we can assume that the prior and posterior PDF of the SNCM $\mathbf{Q}_k$ has a same functional form as the prior and posterior PDF of the one-step PECM $\mathbf{P}_{k|k-1}$ when the estimation error covariance matirx $\mathbf{P}_{k-1|k-1}$ has been determined. According to the VBAKF-PR algorithm, the prior and posterior PDF $\mathbf{P}_{k|k-1}$ are chosen as inverse Wishart (IW) PDF [13].

$$\mathrm{P}(\mathbf{P}_{k|k-1}|\mathbf{Z}_{1:k-1}) = \mathrm{IW}(\mathbf{P}_{k|k-1};\ \hat{\gamma}_{k|k-1},\ \hat{\mathbf{T}}_{k|k-1}) \tag{15}$$

$$\mathrm{P}(\mathbf{P}_{k|k-1}|\mathbf{Z}_{1:k}) = \mathrm{IW}(\mathbf{P}_{k|k-1};\ \hat{\gamma}_{k|k},\ \hat{\mathbf{T}}_{k|k}) \tag{16}$$

where $\hat{\gamma}_{k|k-1}$ and $\hat{\mathbf{T}}_{k|k-1}$ denote the dof parameter and inverse scale matrix of $\mathbf{P}_{k|k-1}$, respectively. Furthermore, where $\hat{\gamma}_{k|k}$ and $\hat{\mathbf{T}}_{k|k}$ denote the dof parameter and inverse scale matrix of the posterior $\hat{\mathbf{P}}_{k|k-1}$ after iterative update, respectively. According to the VBAKF-PR algorithm, the posterior PDF of the one-step PECM $P(\mathbf{P}_{k|k-1}|\mathbf{Z}_{1:k})$ is updated as IW PDF as follows [13]

$$\hat{\gamma}_{k|k} = \hat{\gamma}_{k|k-1} + 1 \tag{17}$$

$$\hat{\mathbf{T}}_{k|k} = \hat{\mathbf{T}}_{k|k-1} + \mathbf{A}_k \tag{18}$$

$$\mathbf{A}_k = \hat{\mathbf{P}}_{k|k} + (\hat{\mathbf{X}}_{k|k} - \hat{\mathbf{X}}_{k|k-1})(\hat{\mathbf{X}}_{k|k} - \hat{\mathbf{X}}_{k|k-1})^T \tag{19}$$

where $\hat{\mathbf{P}}_{k|k}$ denotes the estimation error covariance matrix after iterative update, and $\hat{\mathbf{X}}_{k|k}$ denotes the posterior state estimate after iterative variational update.

Here the prior and posterior PDFs of the SNCM are also selected as an IW PDF based on the assumption that the PDF of the SNCM has the same functional form as the PDF of the one-step PECM, i.e.,

$$\mathrm{P}(\mathbf{Q}_k|\mathbf{Z}_{1:k-1}) = \mathrm{IW}(\mathbf{Q}_k;\ \hat{\mathbf{j}}_{k|k-1},\ \hat{\mathbf{J}}_{k|k-1}) \tag{20}$$

$$\mathrm{P}(\mathbf{Q}_k|\mathbf{Z}_{1:k}) = \mathrm{IW}(\mathbf{Q}_k;\ \hat{\mathbf{j}}_{k|k},\ \hat{\mathbf{J}}_{k|k}) \tag{21}$$

where $\hat{\mathbf{j}}_{k|k-1}$ and $\hat{\mathbf{J}}_{k|k-1}$ denote the dof parameter and inverse scale matrix of $\mathbf{Q}_k$, respectively. The $\hat{\mathbf{j}}_{k|k}$ and $\hat{\mathbf{J}}_{k|k}$ denote the dof parameter and inverse scale matrix of $\mathbf{Q}_k$ after updating according to the measurement information. In this paper, the dof parameter $\hat{\mathbf{j}}_{k|k-1}$ and the inverse scale matrix $\hat{\mathbf{J}}_{k|k-1}$ are propagated from the previous posterior distribution parameters as

$$\hat{\mathbf{j}}_{k|k-1} = r\hat{\mathbf{j}}_{k-1|k-1} \tag{22}$$

$$\hat{\mathbf{J}}_{k|k-1} = r\hat{\mathbf{J}}_{k-1|k-1} \tag{23}$$

where r denote the forgetting factor.

Referring to the updating method of the posterior PDF $P(\mathbf{P}_{k|k-1}|\mathbf{Z}_{1:k})$, the updates of the dof parameter $\hat{\mathbf{j}}_{k|k}$ and inverse scale matrix $\hat{\mathbf{J}}_{k|k}$ are as follows when the iterative update of the one-step

PECM $\hat{\mathbf{P}}_{k|k-1}$ is completed.

$$\hat{\mathbf{j}}_{k|k} = \hat{\mathbf{j}}_{k|k-1} + 1 \tag{24}$$

$$\hat{\mathbf{J}}_{k|k} = \hat{\mathbf{J}}_{k|k-1} + \mathbf{C}_k \tag{25}$$

$$\mathbf{C}_k = \hat{\mathbf{P}}_{k|k-1} - \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^{\mathrm{T}} \tag{26}$$

According to the property of the IW distribution, the modified SNCM is rewritten as [13]

$$\mathbf{Q}_k = \hat{\mathbf{J}}_{k|k}/\hat{\mathbf{j}}_{k|k} \tag{27}$$

The latest estimate of the SNCM $\mathbf{Q}_k$ is substituted into the algorithm in the next iteration. The implementation pseudocode of the proposed VBAKF-PRQ algorithm is shown in Table 1.

Table 1. Implementation pseudocode of the VBAKF-PRQ algorithm

Inputs: $\hat{\mathbf{X}}_{k-1\|k-1}$,$\mathbf{P}_{k-1\|k-1}$,$\hat{\mathbf{j}}_{k-1\|k-1}$,$\hat{\mathbf{J}}_{k-1\|k-1}$,$\hat{\mu}_{k-1\|k-1}$,$\hat{\mathbf{U}}_{k-1\|k-1}$,$r$,$\tau$,$m$,$\mathbf{F}_k$,$\mathbf{H}_k$,$\mathbf{Z}_k$
Perform the time update:
$\hat{\mathbf{X}}_{k\|k-1} = \mathbf{F}_k\hat{\mathbf{X}}_{k-1\|k-1}$
$\mathbf{Q}_{k-1} = \hat{\mathbf{J}}_{k-1\|k-1}/\hat{\mathbf{j}}_{k-1\|k-1}$, $\mathbf{P}_{k\|k-1} = \mathbf{F}_k\mathbf{P}_{k-1\|k-1}\mathbf{F}_k^{\mathrm{T}} + \mathbf{Q}_{k-1}$
Initialization: $\hat{\mathbf{X}}_{k\|k}^{(0)} = \hat{\mathbf{X}}_{k\|k-1}$,$\hat{\mathbf{P}}_{k\|k}^{(0)} = \mathbf{P}_{k\|k-1}$,$\hat{\mathbf{j}}_{k\|k-1} = r\hat{\mathbf{j}}_{k-1\|k-1}$,$\hat{\mathbf{J}}_{k\|k-1} = r\hat{\mathbf{J}}_{k-1\|k-1}$
$\hat{\gamma}_{k\|k-1} = \tau$, $\hat{\mathbf{T}}_{k\|k-1} = \hat{\gamma}_{k\|k-1}\mathbf{P}_{k\|k-1}$
$\hat{\mu}_{k\|k-1} = r\hat{\mu}_{k-1\|k-1}$, $\hat{\mathbf{U}}_{k\|k-1} = r\hat{\mu}_{k\|k-1}\hat{\mathbf{U}}_{k-1\|k-1}$
Perform the measurement update:
for $i = 1 : m$
update $\mathrm{P}(\hat{\mathbf{P}}_{k\|k-1}^{(i)}\|\mathbf{Z}_{1:k}) = \mathrm{IW}(\hat{\mathbf{P}}_{k\|k-1}^{(i)};\ \hat{\gamma}_{k\|k}^{(i)},\ \hat{\mathbf{T}}_{k\|k}^{(i)})$
$\mathbf{A}_k^{(i)} = \hat{\mathbf{P}}_{k\|k}^{(i-1)} + (\hat{\mathbf{X}}_{k\|k}^{(i-1)} - \hat{\mathbf{X}}_{k\|k-1})(\hat{\mathbf{X}}_{k\|k}^{(i-1)} - \hat{\mathbf{X}}_{k\|k-1})^T$
$\hat{\gamma}_{k\|k}^{(i)} = \hat{\gamma}_{k\|k-1} + 1$, $\hat{\mathbf{T}}_{k\|k}^{(i)} = \hat{\mathbf{T}}_{k\|k-1} + \mathbf{A}_k^{(i)}$
update $\mathrm{P}(\hat{\mathbf{R}}_k^{(i)}\|\mathbf{Z}_{1:k}) = \mathrm{IW}(\hat{\mathbf{R}}_k^{(i)};\ \hat{\mu}_{k\|k}^{(i)},\ \hat{\mathbf{U}}_{k\|k}^{(i)})$
$\mathbf{B}_k^{(i)} = \mathbf{H}_k\mathbf{P}_{k\|k}^{(i-1)}\mathbf{H}_k^T + (\mathbf{Z}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k\|k}^{(i-1)})(\mathbf{Z}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k\|k}^{(i-1)})^T$
$\hat{\mu}_{k\|k}^{(i)} = \hat{\mu}_{k\|k-1} + 1$, $\hat{\mathbf{U}}_{k\|k}^{(i)} = \hat{\mathbf{U}}_{k\|k-1} + \mathbf{B}_k^{(i)}$
calculate $\hat{\mathbf{P}}_{k\|k-1}^{(i)} = \hat{\mathbf{T}}_{k\|k}^{(i)}/\hat{\gamma}_{k\|k}^{(i)}$,$\hat{\mathbf{R}}_k^{(i)} = \hat{\mathbf{U}}_{k\|k}^{(i)}/\hat{\mu}_{k\|k}^{(i)}$, $\mathbf{K}_k^{(i)} = \hat{\mathbf{P}}_{k\|k-1}^{(i)}\mathbf{H}_k^{\mathrm{T}}\left(\mathbf{H}_k\hat{\mathbf{P}}_{k\|k-1}^{(i)}\mathbf{H}_k^{\mathrm{T}} + \hat{\mathbf{R}}_k^{(i)}\right)^{-1}$
$\hat{\mathbf{X}}_{k\|k}^{(i)} = \hat{\mathbf{X}}_{k\|k-1} + \mathbf{K}_k^{(i)}\left(\mathbf{Z}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k\|k-1}\right)$, $\hat{\mathbf{P}}_{k\|k}^{(i)} = \hat{\mathbf{P}}_{k\|k-1}^{(i)} - \mathbf{K}_k^{(i)}\mathbf{H}_k\hat{\mathbf{P}}_{k\|k-1}^{(i)}$
end
$\mathbf{C}_k = \hat{\mathbf{P}}_{k\|k-1}^{(m)} - \mathbf{F}\mathbf{P}_{k-1\|k-1}\mathbf{F}^{\mathrm{T}}$
$\hat{\mathbf{j}}_{k\|k} = \hat{\mathbf{j}}_{k\|k-1} + 1$, $\hat{\mathbf{J}}_{k\|k} = \hat{\mathbf{J}}_{k\|k-1} + \mathbf{C}_k$
$\hat{\mathbf{X}}_{k\|k} = \hat{\mathbf{X}}_{k\|k}^{(m)}$,$\mathbf{P}_{k\|k} = \hat{\mathbf{P}}_{k\|k}^{(m)}$,$\hat{\mu}_{k\|k} = \hat{\mu}_{k\|k}^{(m)}$,$\hat{\mathbf{U}}_{k\|k} = \hat{\mathbf{U}}_{k\|k}^{(m)}$
Outputs: $\hat{\mathbf{X}}_{k\|k}$,$\mathbf{P}_{k\|k}$,$\hat{\mathbf{j}}_{k\|k}$,$\hat{\mathbf{J}}_{k\|k}$,$\hat{\mu}_{k\|k}$,$\hat{\mathbf{U}}_{k\|k}$

4 Simulation Results

The performance of the proposed algorithm in SINS/GPS in-motion initial alignment is evaluated by comparing with the existing AKF algorithm. The numerical simulation of SINS/GPS initial alignment is conducted on the computer. The trajectory used in the simulation is shown in Fig. 1, in which the acceleration, uniform, turning and climb are included. The parameter settings of simulation sensors are shown in Table 2. The GPS's three directional lever arms are set to 1 m.

In all the in-motion initial alignment methods, the initial estimation error covariance matrix is set to diag$([(\mathbf{3.0}\times\mathbf{10^{-8}})_{1\times3}\ (\mathbf{9.4}\times\mathbf{10^{-5}})_{1\times3}\ \mathbf{0.0024}_{1\times3}\ \mathbf{1}_{1\times3}])$. The nominal SNCM is chosen as diag$([(\mathbf{6.44}\times\mathbf{10^{-21}})_{1\times3}\ (\mathbf{6.44}\times\mathbf{10^{-19}})_{1\times3}\ \mathbf{0}_{1\times3}\ \mathbf{0}_{1\times3}])$, and the nominal MNCM is chosen as $10\times$ diag$([3\ \ 3\ \ 3]^2)$. The size of the sliding window is set to 100. For the existing VBAKF-PR and the proposed VBAKF-PRQ, the forgetting factor r is set to $1-exp(-6)$ to guarantee the appropriate weight of the MNCM prior information. The tuning parameter τ is chosen as 6 to balance the influence of the one-step PECM and system model information. The averaged RMSE of attitude error is chosen as performance metrics to evaluate the accuracy of initial alignment [9]. The simulation results are shown in Table 3 and Fig. 2–Fig. 6.

Table 2. Parameter settings of sensors in the numerical simulation

Sensors	Constant bias	Random noise	Frequency
Gyroscope	$0.5°/s$	$0.015°/\sqrt{s}$	$100Hz$
Accelerometer	$5mg$	$0.5mg/\sqrt{Hz}$	$100Hz$
GPS	$0m/s$	$0.1m/s$	$100Hz$

Table 3. ARMSEs of attitude angles

Method	OBA	KF	VBAKF-PR	VBAKF-PRQ
Pitch(°)	29.55	1.69	0.34	0.15
Roll(°)	18.93	1.35	0.17	0.05
Heading(°)	54.10	5.38	3.18	0.27

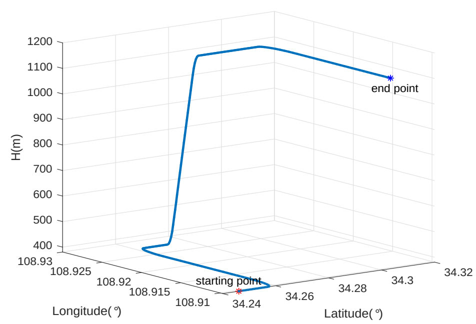

Fig. 1. The plot of the trajectory for the in-motion initial alignment simulation

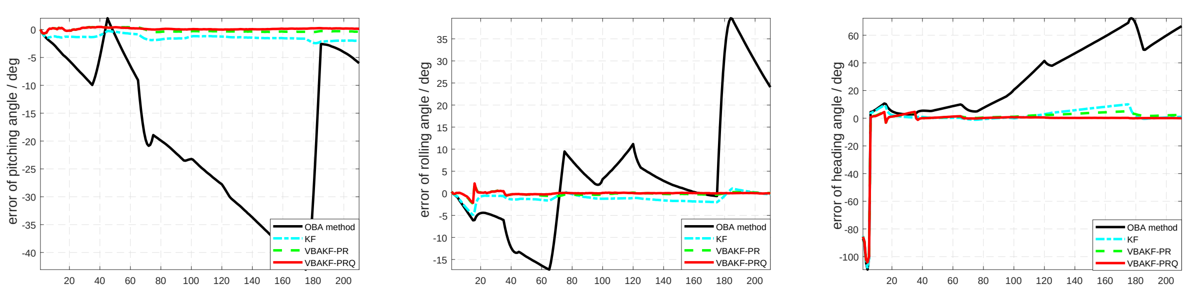

Fig. 2. Plots of the initial alignment attitude estimation errors

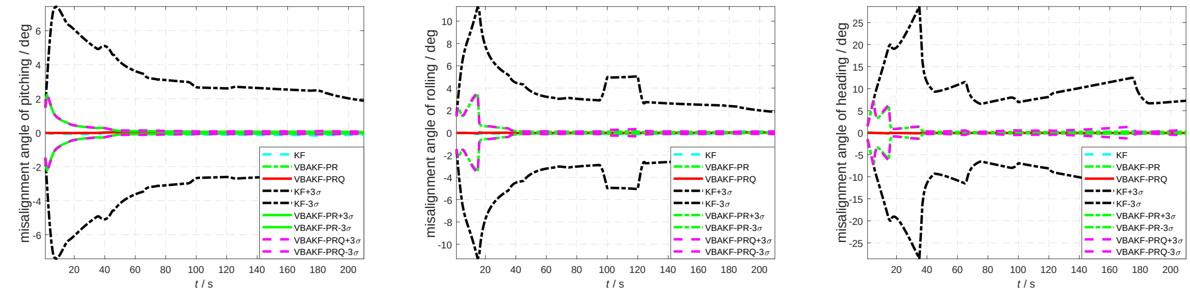

Fig. 3. Plots of the misalignment angle between $\mathbf{C}_{b(t)}^{b(0)}$ and $\hat{\mathbf{C}}_{b(t)}^{b(0)}$

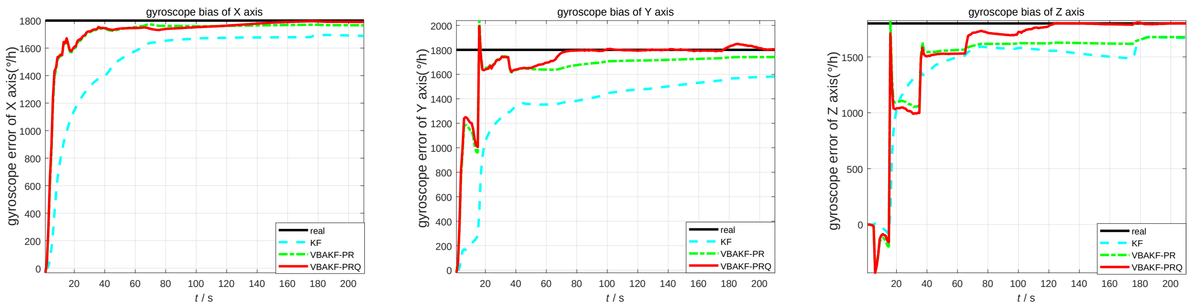

Fig. 4. Plots of the gyroscope drift estimation

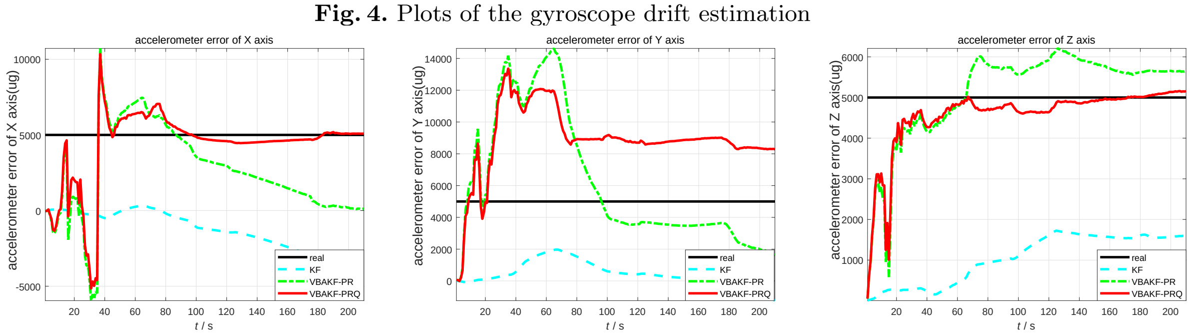

Fig. 5. Plots of the accelerometer bias estimation

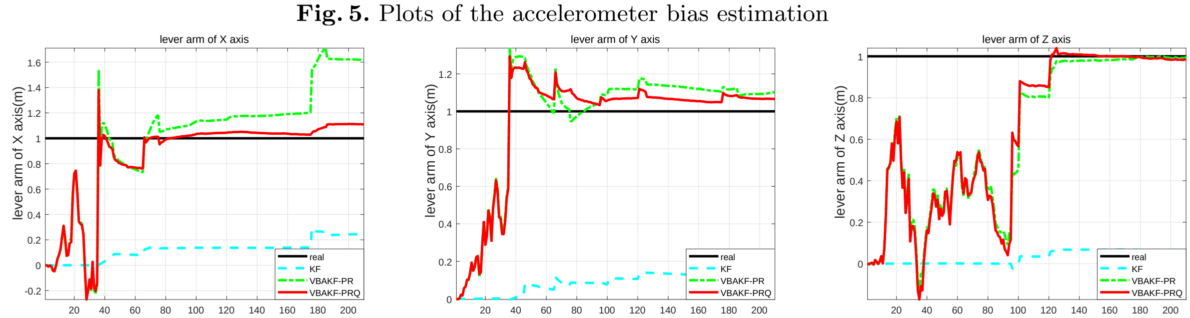

Fig. 6. Plots of the lever arm estimation

In Table 3, it is clear to see that both the VBAKF-PR and VBAKF-PRQ algorithms improve the initial alignment accuracy compared with the traditional Kalman filter, and the initial alignment accuracy of the proposed VBAKF-PRQ algorithm is one order of magnitude higher than the existing VBAKF-PR. This is because the VBAKF-PRQ algorithm not only estimates the unknown MNCM but also updates the unknown SNCM. Fig. 2 shows the attitude estimation error of each algorithm. It is also clear from Fig. 2 that the proposed VBAKF-PRQ is superior to the VBAKF-PR and traditional Kalman filter in terms of initial alignment accuracy, and its estimated results are also more stable. Fig. 3 shows the true misalignment angle between $\mathbf{C}_{b(t)}^{b(0)}$ and $\hat{\mathbf{C}}_{b(t)}^{b(0)}$. It is clear from Fig. 3 that the existing CLAEBIA methods and proposed method solve the divergence problem of attitude matrix $\mathbf{C}_{b(t)}^{b(0)}$ which is caused by error accumulation by estimating and compensating online misalignment angle of the attitude matrix $\mathbf{C}_{b(t)}^{b(0)}$. Furthermore, we choose 3σ in mathematical statistics as the criterion of filter consistency. Here, 3σ is taken as the theoretical error of the misalignment angle. The filter is consistent when the true misalignment angle error is within the $\pm 3\sigma$ range at the time. It is also clear from Fig. 3 that the proposed VBAKF-PRQ algorithm has good consistency.

Fig. 4–Fig. 6 show respectively the estimated and true values of the gyroscope drift, accelerometer bias and lever arm. Fig. 4 shows that the convergence rate of the proposed VBAKF-PRQ algorithm is slightly faster than baseline algorithms. It can also be seen from Fig. 4–Fig. 6 that the estimated values of the proposed algorithm converges to the true values for the gyroscope drift, accelerometer bias and three-axis lever arm, and the proposed method achieves better estimates of sensor bias and lever arm parameters as compared with the traditional Kalman filter method and the VBAKF-PR method. Due to the influence of GPS random noise, it can be seen from Fig.5–Fig.6 that there are some residual errors in the estimates of the accelerometer bias and lever arm. The estimation results of the accelerometer bias and lever arm can be further improved by improving the measurement accuracy of GPS. Moreover, the single-step running time of the Kalman filter, VBAKF-PR and proposed VBAKF-PRQ are 2.1 ms, 2.4 ms and 2.3 ms, respectively, which are all smaller than the sensor sampling period. To sum up, the proposed VBAKF-PRQ improves the initial alignment accuracy considerably and has a moderate amount of computational load.

5 Conclusion and Future Work

In this paper, a novel VBAKF-PRQ algorithm was proposed to solve the state estimation problem with unknown SNCM and MNCM inherent in the linear state-space model of ICLAEIA, in which the state vector, the SNCM and the MNCM are jointly estimated. As the main idea of this paper, the SNCM is assumed to have the same prior and posterior PDFs as the one-step PECM, and then the SNCM can be updated indirectly by using the estimated one-step PECM, since the SNCM cannot be updated directly under the Bayesian estimation framework. Simulation results confirmed that the proposed method is superior to existing methods in initial alignment accuracy. In future work, we will apply the VBAKF-PRQ algorithm to more accurate inertial navigation systems to achieve high-accuracy initial alignment.

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