Removing Speckle Noise by Analysis Dictionary Learning

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***Abstract*—Speckle noise inherently exists in images acquired by coherent systems, for example, synthetic aperture radar (SAR) and sonar images. Removal of speckle noise is a challenging problem because the noise multiplies (rather than adds to) the original image and it does not follow a Gaussian distribution. In this paper, we focus on the speckle noise removal problem and propose a method using analysis dictionary learning. In our proposed method, the image recovery is addressed in the logarithmic transform domain, thereby converting the multiplicative model to an additive model. Our formulation consists of a data fidelity term derived from the distribution of the speckle noise and a regularization term using the learned analysis dictionary. Experimental results on synthetic speckled images and real SAR images demonstrate the promising performance of the proposed method.**

I. Introduction

Speckle noise arises in many images, such as synthetic aperture radar (SAR) and sonar (SAS) images, due to the coherent nature of their acquisition processes. Removing speckle noise is different from the traditional image denoising problem of removing additive Gaussian noise for two reasons. First, speckle noise is multiplicative noise which multiplies (rather than adds to) the original image. Besides, the widely used Gaussian distribution in image denoising is not suitable to describe the statistical properties of speckle noise; the Gamma distribution is one of the most commonly used models for the description of speckle noise [1], [2], [3].

Many methods proposed to remove speckle noise formulate this task as an optimization problem consisting of a data fidelity term and regularization terms. The method of Aubert-Aujol (AA) [4] uses the classical maximum *a posteriori* (MAP) estimate, leading to a data fidelity term and a total variation (TV) regularization term which is applied to the image in the original domain. However, this model is not convex, which raises difficulties from an optimization point of view. In [5], Shi and Osher (SO) eliminate this non-convex issue by applying the TV of the image in the log-domain as the regularization. The multiplicative image denoising by augmented Lagrangian (MIDAL) algorithm [2] addresses the same formulation of SO using a different optimization method, showing advantages in terms of speed and denoising performance. Duran, Fadili and Nikolova (DFN) [1] apply the shrinkage of the curvelet transform coefficients and the TV in the log-domain as the regularization terms. Recently, dictionary learning techniques in sparse representation have also been used to address the speckle noise removal problem. The method proposed in [3] introduces a regularization term using a dictionary learned based on the synthesis model and obtains better denoising results as compared with some existing methods.

In recent years, the analysis model for sparse representation, as a counterpart of the synthesis model, has drawn much attention [6], [7]. For a signal $\mathbf{y} \in \mathbb{R}^m$, this model assumes that the product of $\mathbf{\Omega} \in \mathbb{R}^{p\times m}$ and $\mathbf{y}$ is sparse, i.e. $\mathbf{x} = \mathbf{\Omega}\mathbf{y}$ with $\|\mathbf{x}\|_0 = p - l$, where the ℓ_0-norm $\|\cdot\|_0$ counts the number of non-zero elements of its argument and $0 \leq l \leq p$ is the co-sparsity of $\mathbf{y}$. The matrix $\mathbf{\Omega}$ is usually referred to as an analysis dictionary [8], with each row of $\mathbf{\Omega}$ being an atom. The vector $\mathbf{x} \in \mathbb{R}^p$ is the analysis representation of the signal $\mathbf{y}$ with respect to $\mathbf{\Omega}$. In this model, the analysis dictionary $\mathbf{\Omega}$ plays an important role in the analysis representation of the signal $\mathbf{y}$, and the dictionaries learned from a set of training signals show some advantages compared with pre-defined dictionaries [8]. Some algorithms for learning an analysis dictionary have been proposed [8], [9], [10]. The learned analysis dictionaries have been shown to be useful in denoising additive Gaussian noise, but their employment to removing speckle noise has not been investigated yet.

In this work, we propose a new method for removing speckle noise using analysis dictionary learning. The remainder of the paper is as follows. In Section II, the speckle noise removal task is formulated as a regularized convex optimization problem. Section III introduces our proposed method. The experimental results with synthetic speckled images and real SAR images are presented in Section IV, and Section V concludes the paper.

II. Problem Formulation

Mathematically, the observed image $\mathbf{w} \in \mathbb{R}^N$, contaminated by speckle noise $\mathbf{u} \in \mathbb{R}^N$, can be represented as [2], [3]

$$\mathbf{w} = \mathbf{g} \circ \mathbf{u} \tag{1}$$

where $\mathbf{g} \in \mathbb{R}^N$ denotes the image to be restored. The symbol $\circ$ calculates the Hadamard product (i.e. entry-wise product) of two matrices/vectors. Each entry of $\mathbf{u}$ is assumed to be a random variable following the Gamma distribution whose probability density function is [2], [3], [1]

$$f_u(u) = \frac{L^L}{\Gamma(L)} u^{L-1} e^{-Lu} \tag{2}$$

where L is a positive integer reflecting noise level and $\Gamma(\cdot)$ is the classical Gamma function defined by $\Gamma(L) = (L-1)!$. A smaller L indicates stronger noise.

An additive noise model is obtained by taking the (element-wise) logarithm of both sides of (1), that is

$$\underbrace{\log \mathbf{w}}_{\mathbf{z}} = \underbrace{\log \mathbf{g}}_{\mathbf{y}} + \underbrace{\log \mathbf{u}}_{\mathbf{v}}. \tag{3}$$

The probability distribution of each entry of $\mathbf{v}$ is given by

$$\begin{aligned} f_v(v) &= f_u(e^v) \cdot e^v \\ &= \frac{L^L}{\Gamma(L)} e^{L(v-e^v)}. \end{aligned} \tag{4}$$

Under the independent and identically distributed (i.i.d.) assumption of the entries of $\mathbf{v}$, the probability density of $\mathbf{v}$ is given by

$$f_{\mathbf{v}}(\mathbf{v}) = \prod_{i=1}^{N} \frac{L^L}{\Gamma(L)} e^{L(v_i - e^{v_i})} \tag{5}$$

where v_i denote the entries of the vector $\mathbf{v}$ with $i = 1, 2, ..., N$. Thus, the maximum log-likelihood (ML) estimation of $\mathbf{y}$ is given by the optimal point of the following problem [2]

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}} \sum_{i=1}^{N} (y_i + e^{z_i - y_i}), \tag{6}$$

which is employed as the data fidelity term in our formulation.

We assume log-image $\mathbf{y}$ to be sparse with respect to an analysis dictionary. Since adaptive analysis dictionaries usually have the potential to fit signals better than pre-defined dictionaries [8] as mentioned in Section I, the analysis dictionary learning technique is applied in our proposed method. The details for dictionary learning will be presented in Section III-A. For now, suppose a dictionary $\mathbf{\Omega} \in \mathbb{R}^{p \times m}$ has been learned with log-image patches. In order to apply $\mathbf{\Omega}$ to the restored log-image $\mathbf{y}$, the image patches of the same size are extracted from $\mathbf{y}$ and concatenated as the columns of a matrix $\mathbf{Y} \in \mathbb{R}^{m \times n}$, where n denotes the number of patches. Thus, the co-sparsity of $\mathbf{y}$ can be measured by $\|\mathbf{\Omega Y}\|_0$. Using $\|\mathbf{\Omega Y}\|_0$ as the regularization term, the speckle noise removal problem is formulated as

$$\mathbf{Y}^* = \arg\min_{\mathbf{Y}} \sum_{i=1}^{m} \sum_{j=1}^{n} (\mathbf{Y}_{i,j} + e^{\mathbf{Z}_{i,j} - \mathbf{Y}_{i,j}}) + \lambda \|\mathbf{\Omega Y}\|_0 \tag{7}$$

where λ is the Lagrangian multiplier. The subscript i, j denote the element locating in the ith row and jth column of a matrix. Notice that the matrix $\mathbf{Z} \in \mathbb{R}^{m \times n}$ is obtained with the observed log-image $\mathbf{z}$ by the same approach as generating $\mathbf{Y}$ from $\mathbf{y}$. This formulation (7) combines the data fidelity term obtained via the ML estimation, based on the Gamma distribution, with the regularization term based on the learned dictionary $\mathbf{\Omega}$. However, it is non-convex due to the combinatorial nature of the ℓ_0-norm, which brings about difficulties for optimization. The ℓ_0-norm is thus relaxed as the ℓ_1-norm to construct a convex approximation of (7), that is

$$\mathbf{Y}^* = \arg\min_{\mathbf{Y}} \sum_{i=1}^{m} \sum_{j=1}^{n} (\mathbf{Y}_{i,j} + e^{\mathbf{Z}_{i,j} - \mathbf{Y}_{i,j}}) + \lambda \|\mathbf{\Omega Y}\|_1. \tag{8}$$

This is the formulation of our approach to removing speckle noise based on a learned analysis dictionary.

III. Proposed Method

In general, our proposed method consists of two stages: analysis dictionary learning and image recovery. In the first stage, an analysis dictionary is learned with some image data in the log-domain. The goal of the image recovery stage is to obtain the restored image by solving the optimization problem (8). Our proposed method is referred to as removing speckle noise by analysis dictionary learning (RSN-ADL).

A. Analysis Dictionary Learning Stage

Given a set of training data $\mathbf{A}$, the analysis dictionary learning problem can be written as [11]

$$\begin{aligned} \{\mathbf{\Omega}^*, \mathbf{X}^*\} &= \arg\min_{\{\mathbf{\Omega}, \mathbf{X}\}} \|\mathbf{X} - \mathbf{\Omega A}\|_F^2 \\ &\text{s.t.} \quad \|\mathbf{X}_{:,i}\|_0 = p - l, \ \forall i, \end{aligned} \tag{9}$$

where $\mathbf{X}_{:,i}$ represents the ith column of $\mathbf{X} \in \mathbb{R}^{p \times n}$. This is a general formulation without any additional constraint on $\mathbf{\Omega}$ apart from the co-sparsity constraints $\|\mathbf{X}_{:,i}\|_0 = p - l, \ \forall i$. However, this formulation has ambiguities caused by scaling [10]. In order to avoid these ambiguities, unit ℓ_2-norm constraints on the rows of $\mathbf{\Omega}$ are applied, leading to the following formulation of the Analysis SimCO algorithm [10]

$$\begin{aligned} \{\mathbf{\Omega}^*, \mathbf{X}^*\} &= \arg\min_{\{\mathbf{\Omega}, \mathbf{X}\}} \|\mathbf{X} - \mathbf{\Omega A}\|_F^2 \\ &\text{s.t.} \quad \|\mathbf{X}_{:,i}\|_0 = p - l, \ \forall i \\ &\qquad \|\mathbf{\Omega}_{j,:}\|_2 = 1, \ \forall j, \end{aligned} \tag{10}$$

where $\mathbf{\Omega}_{j,:}$ denotes the jth row of $\mathbf{\Omega}$. The Analysis SimCO algorithm alternates between two stages: analysis sparse coding and dictionary update, as summarized in Algorithm 1.

Algorithm 1 Analysis SimCO

Input: $\mathbf{A}$, p, l

Output: $\mathbf{\Omega}^\star$

Initialization:

Initialize the iteration counter $t = 1$ and the analysis dictionary $\mathbf{\Omega}^{(t)}$. Perform the following steps.

Main Iterations:

1) Analysis sparse coding: Compute the representations $\mathbf{X}^{(t)}$ with the fixed dictionary $\mathbf{\Omega}^{(t)}$ and the training signals in $\mathbf{A}$, based on equations (11) and (12).
2) Dictionary update: Update the dictionary $\mathbf{\Omega}^{(t+1)} \leftarrow \mathbf{\Omega}^{(t)}$, based on equations (14), (15) and (16).
3) If the stopping criterion is satisfied, $\mathbf{\Omega}^\star = \mathbf{\Omega}^{(t+1)}$, quit the iteration. Otherwise, increase the iteration counter $t = t + 1$ and go back to step 1).

The purpose of the analysis sparse coding stage is to get the sparse representations $\mathbf{X}$ of the training signals in $\mathbf{A}$ based on a given dictionary $\mathbf{\Omega}$. The exact representations $\mathbf{X}$ can be calculated directly by simply multiplying the signals in $\mathbf{A}$ by the dictionary $\mathbf{\Omega}$, that is

$$\mathbf{X} = \mathbf{\Omega}\mathbf{A}. \tag{11}$$

Since the initial dictionary is an arbitrary one, the representations obtained in this way may not satisfy the co-sparsity constraints in (10). A hard thresholding operation is therefore applied to enforce the co-sparsity

$$\hat{\mathbf{X}} = HT_l(\mathbf{X}), \tag{12}$$

where $HT_l(\mathbf{X})$ is the non-linear operator that sets the smallest l elements (in magnitude) of each column of $\mathbf{X}$ to zeros. In doing so, the co-sparsity constraints can be enforced.

In the dictionary update stage, $\mathbf{\Omega}$ is updated assuming known and fixed $\mathbf{X}$. In other words, this stage aims at optimizing the following problem

$$\arg\min_{\mathbf{\Omega}} \|\mathbf{X} - \mathbf{\Omega}\mathbf{A}\|_F^2 \quad \text{s.t.} \quad \|\mathbf{\Omega}_{j,:}\|_2 = 1, \ \forall j. \tag{13}$$

Since the Stiefel manifold $\mathcal{S}_{m,1}$ is defined as $\mathcal{S}_{m,1} = \{\mathbf{s} \in \mathbb{R}^m : \mathbf{s}^T\mathbf{s} = 1\}$ [12], the transpose of each row in $\mathbf{\Omega}$ can be seen as one element in $\mathcal{S}_{m,1}$. Thus, the "line" search methods on manifolds can be utilized to deal with problem (13). Here we use the gradient descent line search method on manifolds.

Given that the negative gradient of the objective function (13) with respect to $\mathbf{\Omega}$ is

$$\mathbf{H} = -\frac{\partial\|\mathbf{X} - \mathbf{\Omega}\mathbf{A}\|_F^2}{\partial\mathbf{\Omega}} = 2\mathbf{X}\mathbf{A}^T - 2\mathbf{\Omega}\mathbf{A}\mathbf{A}^T, \tag{14}$$

the search direction of the jth row of $\mathbf{\Omega}$, i.e. the projection of each row of $\mathbf{H}$ onto the tangent space of $\mathcal{S}_{m,1}$, is [12, pp. 49]

$$\bar{\mathbf{h}}_j = \mathbf{H}_{j,:}(\mathbf{I} - \mathbf{\Omega}_{j,:}^T\mathbf{\Omega}_{j,:}). \tag{15}$$

The line search path for the jth row of $\mathbf{\Omega}$ can be written as

$$\mathbf{\Omega}_{j,:}(\alpha) = \begin{cases} \mathbf{\Omega}_{j,:} & \text{if } \|\bar{\mathbf{h}}_j\|_2 = 0, \\ \mathbf{\Omega}_{j,:}\cos(\alpha\|\bar{\mathbf{h}}_j\|_2) + (\bar{\mathbf{h}}_j/\|\bar{\mathbf{h}}_j\|_2)\sin(\alpha\|\bar{\mathbf{h}}_j\|_2) & \\ & \text{otherwise,} \end{cases} \tag{16}$$

where α is the step size. The golden section search method [13] is applied to find a proper step size α.

B. Image Recovery Stage

The aim of the image recovery stage is to remove the speckle noise by addressing the optimization problem (8). This is an ℓ_1 regularized problem, which can be tackled with the alternating direction method of multipliers (ADMM) [14]. In ADMM form, (8) can be written as the following equality-constrained convex optimization problem

$$\begin{aligned} \arg\min_{\mathbf{Y},\mathbf{T}} & \sum_{i=1}^{m}\sum_{j=1}^{n}(\mathbf{Y}_{i,j} + e^{\mathbf{Z}_{i,j} - \mathbf{Y}_{i,j}}) + \lambda\|\mathbf{T}\|_1 \\ & \text{s.t. } \mathbf{T} = \mathbf{\Omega}\mathbf{Y}, \end{aligned} \tag{17}$$

where $\mathbf{T} = \mathbf{\Omega}\mathbf{Y}$. The introduction of the variables in $\mathbf{T}$ is to eliminate the optimization variables in $\mathbf{Y}$ appearing in the ℓ_1 regularization term of (8) and thus make the alternating update of variables possible.

The augmented Lagrangian method is applied to convert (17) to an unconstrained problem. In particular, using a dual parameter $\mathbf{B} \in \mathbb{R}^{p\times n}$, the augmented Lagrangian function for (17) is developed by adding a penalty term $\langle\mathbf{B}, \mathbf{\Omega}\mathbf{Y} - \mathbf{T}\rangle$ and an extra quadratic term related to the constraint $\mathbf{T} = \mathbf{\Omega}\mathbf{Y}$, leading to the new objective function as follows

$$\begin{aligned} L_\gamma(\mathbf{Y}, \mathbf{T}, \mathbf{B}) = & \sum_{i=1}^{m}\sum_{j=1}^{n}(\mathbf{Y}_{i,j} + e^{\mathbf{Z}_{i,j} - \mathbf{Y}_{i,j}}) + \lambda\|\mathbf{T}\|_1 \\ & + \gamma\langle\mathbf{B}, \mathbf{\Omega}\mathbf{Y} - \mathbf{T}\rangle + \frac{\gamma}{2}\|\mathbf{\Omega}\mathbf{Y} - \mathbf{T}\|_F^2 \\ = & \sum_{i=1}^{m}\sum_{j=1}^{n}(\mathbf{Y}_{i,j} + e^{\mathbf{Z}_{i,j} - \mathbf{Y}_{i,j}}) + \lambda\|\mathbf{T}\|_1 \\ & + \frac{\gamma}{2}\|\mathbf{B} + \mathbf{\Omega}\mathbf{Y} - \mathbf{T}\|_F^2 - \frac{\gamma}{2}\|\mathbf{B}\|_F^2, \end{aligned} \tag{18}$$

with $\gamma > 0$ being the penalty coefficient. The ADMM algorithm iteratively updates each of the variables $\{\mathbf{Y}, \mathbf{T}, \mathbf{B}\}$, while keeping the rest fixed. In the t-th iteration, it consists of the following steps

$$\mathbf{Y}^{(t+1)} = \arg\min_{\mathbf{Y}} \ L_\gamma(\mathbf{Y}, \mathbf{T}^{(t)}, \mathbf{B}^{(t)}) \tag{19}$$

$$\mathbf{T}^{(t+1)} = \arg\min_{\mathbf{T}} \ L_\gamma(\mathbf{Y}^{(t+1)}, \mathbf{T}, \mathbf{B}^{(t)}) \tag{20}$$

$$\mathbf{B}^{(t+1)} = \mathbf{B}^{(t)} + (\mathbf{\Omega}\mathbf{Y}^{(t+1)} - \mathbf{T}^{(t+1)}). \tag{21}$$

In fact, herein ADMM can be interpreted as reducing the ℓ_1 regularized problem to solving a sequence of ℓ_2 (squared) regularized problems [14]. For the minimization of (19), the gradient descent method using a fixed step size is applied. For (20), there is the closed-form solution [14]

$$\mathbf{T}^{(t+1)} = ST_{\frac{\lambda}{\gamma}}\{\mathbf{\Omega}\mathbf{Y}^{(t+1)} + \mathbf{B}^{(t)}\}, \tag{22}$$

where $ST_{\frac{\lambda}{\gamma}}$ is the entry-wise soft-thresholding operator defined by

$$ST_{\frac{\lambda}{\gamma}}(\beta) = \begin{cases} \beta - \frac{\lambda}{\gamma}\cdot\text{sgn}(\beta) & \text{if} |\beta| \geq \frac{\lambda}{\gamma}, \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

where $\text{sgn}(\beta)$ returns the sign of β.

The ADMM iterations (19), (20) and (21) are performed until the change of $\mathbf{Y}^{(t+1)}$ is relatively small compared with $\mathbf{Y}^{(t)}$. The restored log-image $\hat{\mathbf{y}}$ can be obtained by reshaping the solution to (17), and thus the denoised image $\hat{\mathbf{g}}$ is obtained by taking the exponential transform of $\hat{\mathbf{y}}$.

IV. Simulation Results

In this section, experiments for synthetic speckled images and real SAR images are presented respectively.

For our proposed RSN-ADL algorithm, the images shown in Fig. 1 were used as the training corpus. Specifically, the training samples employed to learn the analysis dictionary $\mathbf{\Omega}$

were the logarithmic transforms of 20000 patches that were extracted randomly from these corpus images. The size of the training patches was 8×8. The dictionary was initialized as the finite difference operator [6], [8]. The co-sparsity for dictionary learning was set as $l = 100$. The Analysis SimCO algorithm was performed with 2000 iterations.

Fig. 1. Training images used for learning the analysis dictionary.

A. Experiments with Synthetic Speckled Images

We used 3 test images: "Cameraman", "Nîmes" and "Fields", as presented in Fig. 2. These images are commonly used for evaluating the algorithms for removing speckle noise. The size of the Cameraman image is 256×256. The size of Nîmes and Fields is 512×512. The synthetic speckled images were generated by multiplying the pixels of the original images by i.i.d. Gamma random variables (c.f. equations (1) and (2)), with different noise levels $L \in \{10, 4, 1\}$.

Fig. 2. Test images: Cameraman, Nîmes, Fields.

Our proposed method is compared with three recent algorithms: DFN [1] (which outperforms AA [4] and SO [5]), MIDAL [2], and the method proposed in [3] [1]. For description convenience, the method proposed in [3] is referred to as multiplicative noise removal via dictionary learning (MNR-DL) due to the dictionary learning technique involved in the method.

The denoising results are assessed using the peak signal to noise ratio (PSNR), as in [1] and [3]. For a clean image $\mathbf{g} \in \mathbb{R}^N$, the PSNR of its denoised version $\hat{\mathbf{g}} \in \mathbb{R}^N$ is defined as

$$\text{PSNR} = 10 \log_{10} \frac{N|\max(\mathbf{g}) - \min(\mathbf{g})|^2}{\|\hat{\mathbf{g}} - \mathbf{g}\|_2^2} \quad (\text{in dB}) \tag{24}$$

where $\max(\cdot)$ and $\min(\cdot)$ return the maximum value and the minimum value of their operands respectively.

The denoising results are presented in Table I, with bold fonts highlighting the best result in each case. In this table, the results of DFN are obtained from the original paper [1]. The results of MNR-DL for Cameraman and Nîmes with noise levels $L = 10$ and $L = 4$ are also from the original paper [3]. For the results of MNR-DL in other cases, different parameters (i.e. λ in the objective function of MNR-DL [3]) were tested and the best results are reported here. Similarly, the Lagrangian multiplier λ in our model was also tuned by searching roughly to get the highest PSNR. The parameters of MNR-DL and our proposed method are summarized in Table II. For the MIDAL algorithm, its parameters were set at their default values as in [2]. Fig. 3 shows the denoised versions of the Nîmes image with the noise level $L = 1$.

TABLE I
THE DENOISING PSNR RESULTS IN DECIBELS.

L	Algorithm	Cameraman	Nîmes	Fields
10	DFN	26.08	27.80	28.04
	MIDAL	25.40	27.93	27.84
	MNR-DL	**27.32**	**28.85**	28.48
	RSN-ADL	25.39	28.25	**28.83**
4	DFN	22.98	25.84	26.32
	MIDAL	23.42	25.74	26.14
	MNR-DL	**24.91**	**26.34**	26.92
	RSN-ADL	23.47	26.16	**27.15**
1	DFN	19.82	22.64	22.89
	MIDAL	**20.81**	23.03	23.38
	MNR-DL	19.42	22.95	22.20
	RSN-ADL	20.55	**23.67**	**24.05**

From Table I, we can see that our proposed method outperforms the baseline algorithms for the Fields image, in all noise level cases. When $L = 1$, the performance of our algorithm is also the best for the Nîmes image, while MIDAL achieves the best PSNR for Cameraman. In the $L = 4$ cases of the images Cameraman and Nîmes, the denoising results of our method are not as good as that of MNR-DL, but better than the results obtained by DFN and MIDAL.

TABLE II
THE PARAMETERS USED IN THE COMPARISON OF TABLE I.

L	Algorithm	Cameraman	Nîmes	Fields
10	RSN-ADL	0.4	0.3	0.5
	MNR-DL	—	—	3.6
4	RSN-ADL	0.7	0.5	1
	MNR-DL	—	—	0.6
1	RSN-ADL	1.6	1.5	3.5
	MNR-DL	1	3.1	0.01

B. Experiments with Real SAR Images

For this set of experiments, we test our proposed method with some real SAR images[2] as shown in the left column of Fig. 4. The Lagrangian multiplier λ was set as 0.5 for all the SAR images. The denoised images are presented in the right column of Fig. 4. We can see that our proposed method is able to remove speckle noise from the SAR images while keeping

[1]The codes of DFN and MIDAL were downloaded from {https://fadili.users.greyc.fr/software.html} and {http://www.lx.it.pt/~bioucas/publications.html} respectively. We thank the authors of [3] for sharing their code via email.

[2]The test SAR images were downloaded from {https://github.com/zhangyiwei79/Opticks-SAR/tree/master/SAR%20images}

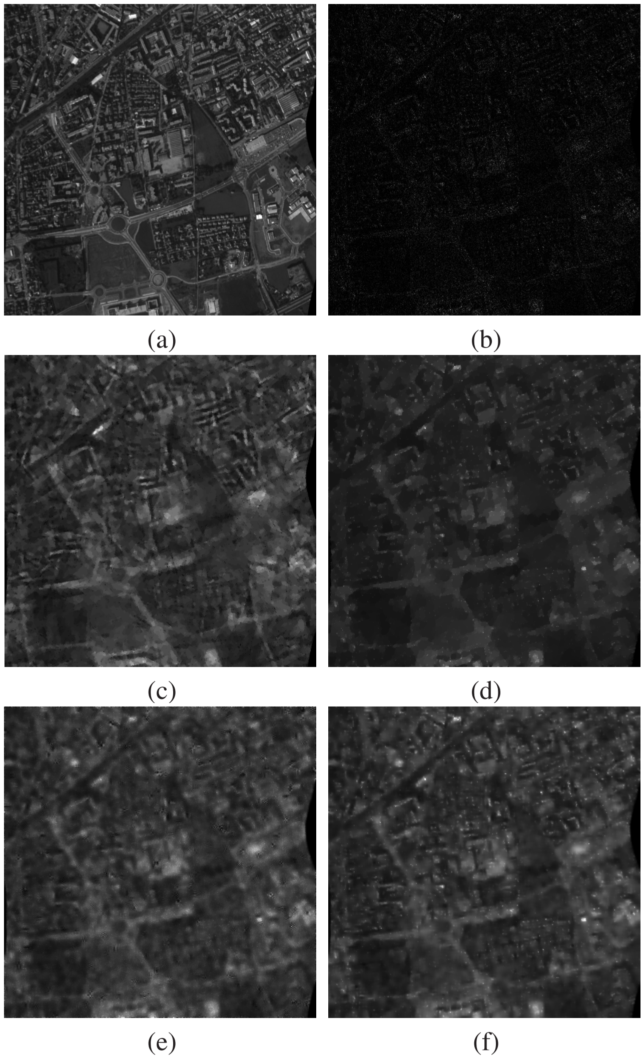

Fig. 3. Results for the Nîmes image (noise level: L=1). (a) Original. (b) Noisy. (c) DFN (22.98 dB). (d) MIDAL (23.03 dB). (e) MNR-DL (22.95 dB). (f) RSN-ADL (23.67 dB).

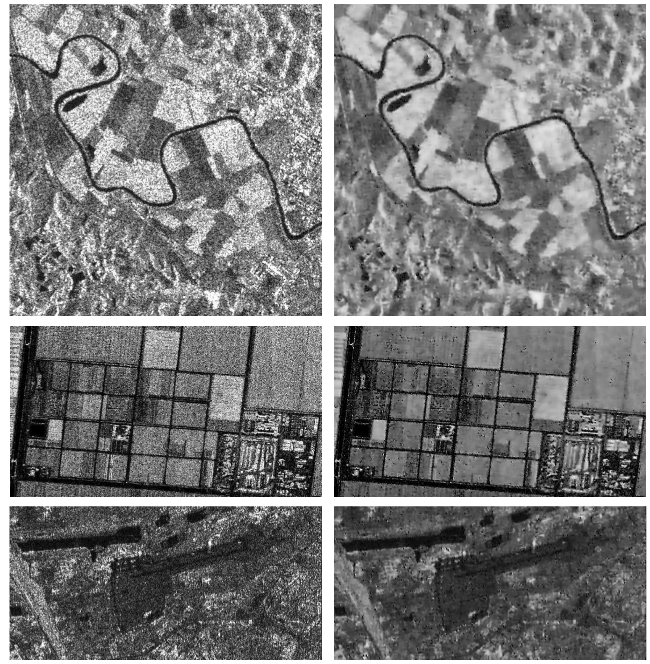

Fig. 4. Original images (left column) and denoised images (right column) obtained by RSN-ADL.

their geometric structures, leading to the denoised images with better visual quality.

V. Conclusion

We have proposed a method for removing speckle noise via analysis dictionary learning. This method addresses the denoising problem in the log-domain, leading to a convex formulation. An analysis dictionary is learned from the logarithmic transforms of some image patches and then this dictionary is employed in a regularization term for restoring the image in the log-domain. The optimization is addressed with ADMM. Simulation results with synthetic images and real SAR images demonstrate the encouraging performance of our proposed method.

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