Fisher Information Matrix Constrained Joint Array and Spatial Sparsity Optimisation for DoA Estimation

By Mingyang Chen, Wenwu Wang

Emails: {m.chen; w.wang}@surrey.ac.uk
Department of Electronic Engineering
University of Surrey, Guildford GU2 7XH, United Kingdom

Abstract

Recently, a joint array and spatial sparsity approach has been presented for direction of arrival (DoA) estimation. Here, we propose a Fisher Information Matrix (FIM) constrained version of this method. In the step of sparse array optimisation, a constraint with FIM is considered to reduce the error before scaling the observed signal by the weight coefficients, followed by the spatial sparsity reconstruction, where the difference between the reconstruted result and the desired beam response is constrained by a statistic expression. The simulation results show that some small improvements can be obtained by imposing the proposed method.

1. Introduction

In acoustic environments, one of the most important tasks is to localise the source signal. Direction of arrival (DoA) estimation based on array signal processing has attracted great interests in applications, such as underwater acoustic detection, target tracking and environmental monitoring [1] [2]. Traditionally, Capon beamformer, high-resolution and multiple signal classification (MUSIC) algorithm are three main methods for DoA estimation [3] [4] [5]. Recently, Spatial sparse representation (SSR) has been addressed to reconstruct source signal by extracting meaningful lower-dimensional information from high-dimensional data [6]. A typical implementation for SSR is to use compressive sensing (CS) [7], where the activity of source is assumed to be sparse and the sparsity is enforced by a constraint based on l_1 norm of a vector of the coefficients corresponding to the source activities in the spatial domain [8].

Previously our work [9] has proposed a joint optimisation of sparse array and spatial sparsity, to achieve source detection in a subset of space with as few sensors as possible. The benefits of using partial array sensors include reducing the cost for array design and manufacturing, limiting the sensor storage and physical space, and countering against sensor failure.

In this paper we impose statistic constraints on the existing joint approach, where the Fisher Information Matrix is used to express the Maximum Likelihood Estimation (MLE) of the source signal [10] [11]. Through limiting the difference between the reconstruction result and the MLE of desired response, the performance of sparse optimisation can be improved .

2. Background

2.1. *Signal Model*

We assume that a narrowband source signal arrives in one half of the plane and the array is expected to have a perfect baffle, which means the arrival directions are from -90 degrees to +90 degrees along the plane of the array elements, reflecting as a vector for the DoA $\mathbf{x}_k = (x_{k1}, x_{k2}, \cdots, x_{kM})^{-1}$, where M is the number of potential source directions and 0 degree is the normal to the line of the array. At each time step k, the observed signal is denoted as $\mathbf{y}_k = (y_{k1}, y_{k2}, \cdots, y_{kN})^{-1}$, where N is the number of potential sensors. A linear array is used and each array element is supposed to have an equivalent sensitivity.

The dictionary matrix $\mathbf{A}$ to describe the possible global source directions is created with the size of $\mathbf{A} \in \mathbb{C}^{N\times M}$, where $N \ll M$. The nm-th element of $\mathbf{A}$ is defined by

$$\mathbf{A}_{nm} = \frac{1}{\sqrt{N}} exp\left[-j2\pi\mu_n \sin\theta_m\right] \tag{2.1}$$

where $j \equiv \sqrt{-1}$, $\mu_n = \dfrac{d_n}{cT_s}$ and T_s stands for the sampling period at $n = 1, 2, \cdots, N$, d_n denotes the distance between the n-th sensor and the middle sensor, c is the speed of wave propagation, and $\theta_m = \frac{\pi m}{M} - \pi/2$ is the DoA of the m-th hypothetical source to the n-th sensor in the array.

Therefore, the N dimensional source signal can be defined as the array model

$$\mathbf{y}_k = \mathbf{A}\mathbf{x}_k + \mathbf{n}_k \tag{2.2}$$

where $\mathbf{n}_k$ is the vector of random noise produced at each k. Here we consider isotropic noise in the assumed half plane as used in our experiments.

2.2. *Fisher Information Matrix*

In statistics, the Fisher Information Matrix (FIM) [11], which is a method to calculate the Maximum Likelihood Estimation (MLE), can be defined as follow.

$$\mathbf{F}(\mathbf{x}) = \mathbb{E}\{(\frac{\partial log\mathbf{y}(\mathbf{x})}{\partial \mathbf{x}})(\frac{\partial log\mathbf{y}(\mathbf{x})}{\partial \mathbf{x}})^T\} \tag{2.3}$$

where $(\cdot)^T$ denotes the transpose of a matrix.

3. FIM Constrained Joint Sparse Optimisation

According to our recent work [9] [12], the CS-based narrowband array optimisation for the sparse complex vector weight coefficients, i.e. $\mathbf{w} = \mathbf{w}_R + \mathbf{w}_I j$, can be solved by minimising the l_1 norm of the weight coefficients [13]. A FIM constraint here is used to limit the difference between the MLE of $\mathbf{y}_k$ and the scaled $diag(\mathbf{w})\mathbf{y}_k$, so that in the next step $\mathbf{y}_k$ can be replaced by $diag(\mathbf{w})\mathbf{y}_k$ more strictly. The constrained l_1 norm is written as below.

$$\begin{aligned} \min \quad & \|\mathbf{w}_R\|_1 + \|\mathbf{w}_I\|_1 \\ \text{subject to} \quad & \|\mathbf{p} - (\mathbf{w}_R + \mathbf{w}_I j)^H \mathbf{A}\|_2 \leq \alpha \\ & \|\mathbf{f}(\mathbf{x}) - diag(\mathbf{w})\mathbf{y}\|_1 \leq \beta N \end{aligned} \tag{3.1}$$

where $\mathbf{p} \in \mathbb{C}^M$ is the vector holding the desired beam response at the sampled angular points θ_m for the frequency of interest Ω. To be specific, $\mathbf{p} = [p(\Omega, \theta_1), \cdots, p(\Omega, \theta_M)]$, where $p(\Omega, \theta)$ is a desired response at the direction θ and frequency Ω. $\mathbf{f}(\mathbf{x})$ is the $\mathbf{N}$ dimensional vector holding the values of each element in the column of $\mathbf{F}(\mathbf{x})^T$ and the expectation is chosen by calculating the average values, $\alpha \in \Re^+$ is a threshold measuring

Implementation of statistic constrained joint sparse approach
Input: observed signal $\mathbf{y}_k$ Output: weight coefficients for sensors: $\mathbf{w}$ spatial sparsity: $\mathbf{x}_k$ and estimated DoA: $\mathbf{p}$ Initialisation: generate $\mathbf{p} \in \mathbb{C}^{1\times M}$ at random degrees, Run: for $k = 1, 2, 3, \cdots$ for $kk = 1, 2$ optimise (3.1) to obtain $\mathbf{w}_R$ and $\mathbf{w}_I$ $\mathbf{w} = \mathbf{w}_R + \mathbf{w}_I i$ obtain $\mathbf{D}$ as in (3.4) form $\mathbf{W}$ as in (3.3) optimise (3.2) to obtain $\mathbf{x}_k$ reconstruct $\mathbf{p}$ as in (3.5) $\mathbf{p}_{reconstruct} = \mathbf{p}$ end end

TABLE 1. The proposed statistic constrained joint sparse approach.

the similarity between the designed response and the desired response, $\beta \in \Re^{+}$ is also a threshold, $(\cdot)^H$ is a Hermitian operator, $\|\cdot\|_1$ and $\|\cdot\|_2$ are respectively the ℓ_1 and ℓ_2 norm of their arguments.

With the result of estimated $\mathbf{w}$ in (3.1), the spatial sparsity based DoA estimation can be achieved by a sequential Bayesian technique based on the least absolute shrinkage and selection operator (LASSO) algorithm [14] [15], where the input of narrowband signal $\mathbf{y}_k$ at time k can be scaled [9]. A constraint to improve the performance of reconstruction is added to the LASSO function.

$$\begin{aligned} \underset{\mathbf{x}_k, v_m}{\text{argmin}} \quad & \|\mathbf{W}\mathbf{y}_k - \mathbf{A}\mathbf{x}_k\|_2^2 + \mu\|\mathbf{D}\mathbf{x}_k\|_1 \\ \text{subject to} \quad & \|\mathbf{f}(\mathbf{x})^T\mathbf{A} - \mathbf{p}_{reconstruct}\|_1 \leq \gamma M \end{aligned} \tag{3.2}$$

with

$$\mathbf{W} = diag(|\mathbf{w}|) \tag{3.3}$$

$$\mathbf{D} = \sigma^2\mathbf{V}, \quad \mathbf{V} = diag(\mathbf{v}) \tag{3.4}$$

$$\mathbf{p}_{reconstruct} = (\mathbf{A}\mathbf{x}_k)^H\mathbf{A} \tag{3.5}$$

where $\mathbf{D}$ and $\mathbf{V}$ are the matrices holding the coefficients vector $\mathbf{v} = (v_1, v_2, \ldots, v_{\mathrm{M}})^T$, which corresponds to the source activity in the source space, μ is a regularization parameter, and σ^2 is the noise variance [15]. The value of MLE is used to constraint the similarity between the DoA reconstruction and the expression that MLE of $\mathbf{y}_k$ maps on the matrix $\mathbf{A}$, so that the DoA estimation can be more robust. Similar to β, $\gamma \in \Re^{+}$ is a constrained parameter.

Both cost functions (3.1) and (3.2) are implemented by the CVX toolbox in Matlab [16]. The alternating procedure of the statistic constrained joint sparse optimisation is presented as Table, where the initial input beam response $\mathbf{p}$ can be set randomly.

4. Numerical Simulations

In this section, the proposed FIM constrained joint sparse optimisation for DoA estimation is performed on a simulated narrowband moving source signal, which starts from 50 degrees and decreases uniformly to -49 degrees. The intialised input beam response is

	MSE_{array} (dB)	$MSE_{spatial}$ (dB)	Active sensors	SNR (dB)
Baseline	-64.56	-13.99	20.78	-
FIM constrained	-64.56	-13.99	20.78	-
Baseline	-54.36	-12.36	20.76	20
FIM constrained	-56.70	-12.93	20.76	20
Baseline	-62.94	-14.43	20.75	30
FIM constrained	-63.11	-14.43	20.73	30
Baseline	-64.20	-15.47	20.80	40
FIM constrained	-64.49	-14.25	20.76	40

TABLE 2. Comparison between the proposed FIM constrained method and the baseline method without FIM constraints.

at 20 degrees. We assume the source direction is changing at a constant range, hence no Doppler shift is considered in this paper.

The underwater speed of sound used in this model is set to be 1500 m/s, and the frequency of the sources is 200 Hz. A linear array with the grid of 100 potential sensors is used. The seperation distance between adjacent sensors is 0.05λ (λ is the wavelength) and the maximum running step is $K = 100$. The inter-sensor spacing and length of running time are chosen according to the number of sensors and the complexity of source signals to ensure convergence. The constraint value of α used in (3.1) is 0.3, the value of β is 0.3 and the value of γ in (3.2) is 0.1. A series of experiments for the baseline method without the FIM constraints are also tested.

The performance index in terms of Mean Square Errors (MSEs) for sparse array optimisation and spatial sparsity optimisation are used according to functions (3.1) and (3.2) as

$$MSE_{array} = 20log_{10}\left(\frac{\|\mathbf{p} - \mathbf{w}^H\mathbf{A}\|_2^2}{M}\right) dB, MSE_{spatial} = 20log_{10}\left(\frac{\|\mathbf{y}_k^H\mathbf{A} - \mathbf{p}\|_2^2}{M}\right) dB \quad (4.1)$$

where M is the number of potential source directions. Both MSEs are measured along the time length K and the average value is calculated.

From Table 2, it can be observed that for the moving source, the FIM constrained joint sparse optimisation gives a satisfatory performance. With the increase of noise level (Signal to Noise Ratio (SNR)), the DoA estimations become more similar to the desired beam response. The FIM constrained method offers small improvements over the baseline method.

5. Conclusion and Future Work

This paper considers an idea of adding FIM constraints to the two-step joint array and spatial sparsity optimisation. The constraints are used to reduce the error when scaling the observed signal and to limit the difference between the reconstruction result and the desired beam response. The results show the potential of the proposed FIM constrained method, which worths further investigation in our future work.

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